

Mass and Decay Constant of the $D_2^*(2460)$ Tensor Meson

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Abstract

We calculate the mass and decay constant of the $D_2^*(2460)$ tensor meson in the framework of QCD sum rules. The obtained result on the mass is compatible with the experimental data. Our prediction on the decay constant can be checked in experiment.

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1 Introduction

The semileptonic B meson transitions to the orbitally excited charmed mesons as well as the strong transitions of the excited charmed mesons into the other charmed states have been in focus of attention of many Collaborations in the recent years. The BaBar and Belle Collaborations reported the measurement of the products/ratios of the branching fractions of some semileptonic and hadronic decays of the B meson and orbitally excited charmed meson channels [1–3]. Considering these experimental progress and the fact that the decay channels containing orbitally excited charmed meson in the final state supply a considerable contribution to the total semileptonic B meson decay width, more knowledge about the properties of orbitally excited charmed mesons like $D_2^*(2460)$ is needed both experimentally and theoretically.

In the present letter, we calculate the mass and decay constant of the $D_2^*(2460)$ tensor meson with quantum numbers $I(J^P) = 1/2 (2^+)$ in the framework of the QCD sum rules as one of the most applicable and powerful non-perturbative approaches to hadron physics. For details about the method and some of its applications see for instance [4–7]. Our results on the properties of $D_2^*(2460)$ tensor meson can be used in theoretical calculations on the decays of heavy mesons to orbitally excited charmed $D_2^*(2460)$ meson or semileptonic and strong decay channels of the $D_2^*(2460)$ into other charmed or lighter mesons which may be done to verify the experimental data. Some properties such as mass, decay constant and electromagnetic multi-poles of the heavy-heavy, light-heavy and light-light tensor mesons have previously calculated using different frameworks. For some of them see [8–13] and references therein. Some semileptonic decays of the B meson into the orbitally excited charmed mesons have also been studied in [14] within the framework of constituent quark model.

The letter is organized as follows. In section II, we briefly present calculations of the mass and decay constant of the $D_2^*(2460)$ tensor meson within the framework of the QCD sum rules. Section III is devoted to the numerical analysis of the considered observables as well as comparison of our result on mass with experimental data.

2 QCD sum rules for mass and decay constant of the $D_2^*(2460)$ tensor meson

This section is dedicated to calculation of the mass and decay constant of the $D_2^*(2460)$ tensor meson in the framework of the QCD sum rules. The starting point is to consider the following two-point correlation function:

$$\Pi_{\mu\nu,\alpha\beta} = i \int d^4x e^{iq(x-y)} \langle 0 | \mathcal{T}[j_{\mu\nu}(x) \bar{j}_{\alpha\beta}(y)] | 0 \rangle, \quad (1)$$

where, $j_{\mu\nu}$ is the interpolating current of the $D_2^*(2460)$ tensor meson and \mathcal{T} is the time ordering operator. The current $j_{\mu\nu}$ is written in terms of the quark fields as

$$j_{\mu\nu}(x) = \frac{i}{2} \left[\bar{u}(x) \gamma_\mu \overleftrightarrow{D}_\nu(x) c(x) + \bar{u}(x) \gamma_\nu \overleftrightarrow{D}_\mu(x) c(x) \right], \quad (2)$$

where the $\overleftrightarrow{\mathcal{D}}_\mu(x)$ denotes the four-derivative with respect to x acting on the left and right, simultaneously. It is given as

$$\overleftrightarrow{\mathcal{D}}_\mu(x) = \frac{1}{2} \left[\overrightarrow{\mathcal{D}}_\mu(x) - \overleftarrow{\mathcal{D}}_\mu(x) \right], \quad (3)$$

with,

$$\begin{aligned} \overrightarrow{\mathcal{D}}_\mu(x) &= \overrightarrow{\partial}_\mu(x) - i\frac{g}{2}\lambda^a A_\mu^a(x), \\ \overleftarrow{\mathcal{D}}_\mu(x) &= \overleftarrow{\partial}_\mu(x) + i\frac{g}{2}\lambda^a A_\mu^a(x). \end{aligned} \quad (4)$$

Here, λ^a are the Gell-Mann matrices and $A_\mu^a(x)$ is the external gluon fields. Considering the Fock-Schwinger gauge ($x^\mu A_\mu^a(x) = 0$), these fields are expressed in terms of the gluon field strength tensor

$$A_\mu^a(x) = \int_0^1 d\alpha \alpha x_\beta G_{\beta\mu}^a(\alpha x) = \frac{1}{2}x_\beta G_{\beta\mu}^a(0) + \frac{1}{3}x_\eta x_\beta \mathcal{D}_\eta G_{\beta\mu}^a(0) + \dots \quad (5)$$

The currents contain derivatives with respect to the space-time, hence we consider the two currents at points x and y . After applying the derivatives with respect to the y , we will put $y = 0$.

According to the general criteria of the QCD sum rules, the aforementioned correlation function is calculated via two alternative ways: phenomenologically (physical side) and theoretically (QCD side). In physical side, the two-point correlation function is calculated in terms of the hadronic degrees of freedom like mass, decay constant, etc. The QCD side is obtained in terms of the QCD parameters such as quark masses, quark and gluon condensates, etc. The two-point QCD sum rules is obtained matching coefficients of the same structure representing the tensor mesons from both sides through a dispersion relation and quark-hadron duality assumption. Finally, we apply Borel transformation to stamp down the contributions belong to the higher states and continuum.

2.1 The physical side

In the physical side, the correlation function is obtained inserting complete set of hadronic state having the same quantum numbers as the interpolating current $j_{\mu\nu}$ into Eq. (1). After performing integral over four- x and putting $y = 0$, we obtain the physical side of correlation function as following form:

$$\Pi_{\mu\nu,\alpha\beta} = \frac{\langle 0 | j_{\mu\nu}(0) | D_2^*(2460) \rangle \langle D_2^*(2460) | j_{\alpha\beta}(0) | 0 \rangle}{m_{D_2^*(2460)}^2 - q^2} + \dots, \quad (6)$$

where \dots represents contribution of the higher states and continuum. To proceed, we need to know the matrix element $\langle 0 | j_{\mu\nu}(0) | D_2^*(2460) \rangle$, which is defined in terms of the decay constant, mass and polarization tensor

$$\langle 0 | j_{\mu\nu}(0) | D_2^*(2460) \rangle = f_{D_2^*(2460)} m_{D_2^*(2460)}^3 \varepsilon_{\mu\nu}. \quad (7)$$

Combining Eq. (6) and Eq. (7) and performing summation over polarization tensor via

$$\varepsilon_{\mu\nu}\varepsilon_{\alpha\beta}^* = \frac{1}{2}T_{\mu\alpha}T_{\nu\beta} + \frac{1}{2}T_{\mu\beta}T_{\nu\alpha} - \frac{1}{3}T_{\mu\nu}T_{\alpha\beta}, \quad (8)$$

with,

$$T_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{D_2^*(2460)}^2}, \quad (9)$$

the final representation of physical side is obtained

$$\Pi_{\mu\nu,\alpha\beta} = \frac{f_{D_2^*(2460)}^2 m_{D_2^*(2460)}^6}{m_{D_2^*(2460)}^2 - q^2} \left\{ \frac{1}{2}(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha}) \right\} + \text{other structures} + \dots, \quad (10)$$

where, the explicitly written structure gives contribution to the tensor state.

2.2 The QCD side

The correlation in QCD side, is calculated in deep Euclidean region, $q^2 \ll 0$, by the help of operator product expansion (OPE) where the short and long distance contributions are separated. The short distance effect is calculated using the perturbation theory, while the long distance effect is parameterized in terms of quark and gluon condensates.

Any coefficient of the structure, $\{\frac{1}{2}(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha})\}$, in QCD side, i.e. $\Pi(q^2)$, can be written as a dispersion relation

$$\Pi(q^2) = \int ds \frac{\rho(s)}{s - q^2}, \quad (11)$$

where the spectral density is given by the imaginary part of the $\Pi(q^2)$ function: $\rho(s) = \frac{1}{\pi} \text{Im}[\Pi(s)]$. As we mentioned above, the correlation function contains both perturbative and non-perturbative effects, hence the spectral density can be decomposed as

$$\rho(s) = \rho^{\text{pert}}(s) + \rho^{\text{nonpert}}(s), \quad (12)$$

where, $\rho^{\text{pert}}(s)$ and $\rho^{\text{nonpert}}(s)$ denote the contributions coming from perturbative and non-perturbative effects, respectively.

Now, we proceed to calculate the spectral density $\rho(s)$. Making use of the tensor current presented in Eq. (2) into the correlation function in Eq. (1) and contracting out all quark fields applying the Wick's theorem, we get:

$$\begin{aligned} \Pi_{\mu\nu,\alpha\beta} = & \frac{i}{4} \int d^4x e^{iq(x-y)} \left\{ \text{Tr} \left[S_u(y-x) \gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu(x) \overleftrightarrow{\mathcal{D}}_\beta(y) S_c(x-y) \gamma_\alpha \right] \right. \\ & \left. + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \right\}. \end{aligned} \quad (13)$$

To obtain the correlation function from QCD side, we need to know the heavy and light quarks propagators $S_c(x-y)$ and $S_u(x-y)$. These propagators have been calculated in

[6]. Ignoring the gluon fields which have very small contributions in our calculations (see also [8, 9]), their explicit expressions between two points up to quark condensates can be written as

$$S_c^{ij}(x-y) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot (x-y)} \left\{ \frac{\not{k} + m_c}{k^2 - m_c^2} \delta_{ij} + \dots \right\}, \quad (14)$$

and

$$\begin{aligned} S_u^{ij}(x-y) &= i \frac{\not{x} - \not{y}}{2\pi^2(x-y)^4} \delta_{ij} - \frac{m_u}{4\pi^2(x-y)^2} \delta_{ij} - \frac{\langle \bar{u}u \rangle}{12} \left[1 - i \frac{m_u}{4} (\not{x} - \not{y}) \right] \delta_{ij} \\ &- \frac{(x-y)^2}{192} m_0^2 \langle \bar{u}u \rangle \left[1 - i \frac{m_u}{6} (\not{x} - \not{y}) \right] \delta_{ij} + \dots \end{aligned} \quad (15)$$

Note that the gluon condensates are also ignored in [15–17] since their contributions are suppressed by large denominators, so they play minor roles in calculations.

The next step is to put the expressions of the propagators and apply the derivatives with respect to x and y in Eq. (13) and finally set $y = 0$. As a result, the following expression for the QCD side of the correlation function in coordinate space is obtained:

$$\Pi_{\mu\nu,\alpha\beta} = \frac{N_c}{16} \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{k^2 - m_c^2} \int d^4x e^{iq \cdot x} \{ [Tr \Gamma_{\mu\nu,\alpha\beta}] + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \}, \quad (16)$$

where $N_c = 3$ is the color factor and ,

$$\begin{aligned} \Gamma_{\mu\nu,\alpha\beta} &= k_\nu k_\beta \left[\frac{i \not{x}}{2\pi^2 x^4} + \left(\frac{1}{12} + \frac{x^2}{192} m_0^2 \right) \langle \bar{u}u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha \\ &- ik_\nu \left[\frac{i}{2\pi^2} \left(\frac{\gamma_\beta}{x^4} - \frac{4x_\beta \not{x}}{x^6} \right) + \frac{x_\beta}{96} m_0^2 \langle \bar{u}u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha \\ &+ ik_\beta \left[\frac{i}{2\pi^2} \left(\frac{4x_\nu \not{x}}{x^6} - \frac{\gamma_\nu}{x^4} \right) + \frac{x_\nu}{96} m_0^2 \langle \bar{u}u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha \\ &+ \left[\frac{8i}{2\pi^2} \left(\frac{x_\beta x_\nu \not{x}}{x^8} + \frac{1}{2x^6} \left(\delta_\beta^\nu \not{x} - \gamma_\nu x_\beta + \gamma_\beta x_\nu \right) + \frac{\gamma_\nu x_\beta}{x^6} - \frac{4x_\beta x_\nu \not{x}}{x^8} \right) \right. \\ &\left. + \frac{\delta_\beta^\nu}{96} m_0^2 \langle \bar{u}u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu], \end{aligned} \quad (17)$$

After performing all traces in Eq. (16), in order to calculate the integrals, first we transform the $\frac{1}{(x^2)^n}$ terms to the momentum space ($x \rightarrow p$) and replace $x_\mu \rightarrow -i \frac{\partial}{\partial q_\mu}$. The integral over four- x gives us the Dirac Delta function and using this function, we perform the integral over four- k . The last integral which is over four- p is performed using the Feynman parametrization and the relation

$$\int d^4p \frac{(p^2)^\beta}{(p^2 + L)^\alpha} = \frac{i\pi^2 (-1)^{\beta-\alpha} \Gamma(\beta+2) \Gamma(\alpha-\beta-2)}{\Gamma(2) \Gamma(\alpha) [-L]^{\alpha-\beta-2}}. \quad (18)$$

After dimensional regularization and taking the imaginary part and selecting the coefficient of the aforesaid structure, the spectral densities are obtained as:

$$\begin{aligned} \rho^{pert}(s) &= \frac{N_c}{2^{10} 3^2 \pi^2 s^3} \left[18m_c^{10} - 19m_c^8 s - 116m_c^6 s^2 - 132m_c^4 s^3 + 130m_c^2 s^4 + 119s^5 \right. \\ &\left. + 36m_c^2 s^3 \left(5m_c^2 + 8s \right) \log\left[\frac{m_c^2}{s}\right] \right], \end{aligned} \quad (19)$$

and

$$\rho^{nonpert}(s) = -\frac{N_c}{48s} m_c m_0^2 \langle \bar{u}u \rangle, \quad (20)$$

After achieving the correlation function in two different ways, now we match these two different representations to obtain two-point QCD sum rules for the decay constant and mass of the $D_2^*(2460)$ tensor meson. In order to suppress contribution of the higher states and continuum, we apply Borel transformation with respect to the initial momentum squared, q^2 , to both sides of the sum rules and use the quark-hadron duality assumption. As a result, the sum rule for the leptonic decay constant of the $D_2^*(2460)$ tensor meson is obtained as

$$f_{D_2^*(2460)}^2 e^{-m_{D_2^*(2460)}^2/M^2} = \frac{1}{m_{D_2^*(2460)}^6} \int_{m_c^2}^{s_0} ds \left(\rho^{pert}(s) + \rho^{nonpert}(s) \right) e^{-s/M^2}, \quad (21)$$

where s_0 is the continuum threshold and M^2 is the Borel mass parameter. Differentiating Eq. (21) with respect to $-\frac{1}{M^2}$, then dividing both sides of the obtained result to both sides of Eq. (21), we obtain the sum rule for the mass of the $D_2^*(2460)$ tensor meson

$$m_{D_2^*(2460)}^2 = \frac{\int_{m_c^2}^{s_0} ds \left(\rho^{pert}(s) + \rho^{nonpert}(s) \right) s e^{-s/M^2}}{\int_{m_c^2}^{s_0} ds \left(\rho^{pert}(s) + \rho^{nonpert}(s) \right) e^{-s/M^2}}. \quad (22)$$

3 Numerical results

In this section, we carry out the numerical analysis of the sum rules for the mass and decay constant of the $D_2^*(2460)$ tensor meson. The input parameters are taken to be the values $m_c = (1.27_{-0.09}^{+0.07}) \text{ GeV}$ [18], $\langle \bar{u}u(1 \text{ GeV}) \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3$ [19] and $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$ [20]. The sum rules contain also two auxiliary parameters, namely the Borel parameter M^2 and continuum threshold s_0 . According to the standard procedure in QCD sum rules, the physical quantities should be independent of these parameters, hence we shall look for working regions for these parameters such that the mass and decay constant of the $D_2^*(2460)$ tensor meson weakly depend on these helping parameters. The continuum threshold s_0 is not totally arbitrary and it is in correlation with the energy of the first excited state. We choose the interval $s_0 = (8.1 \pm 0.5) \text{ GeV}^2$ for the continuum threshold. The working region for the Borel mass parameter is determined requiring that not only the higher state and continuum contributions are suppressed but also the contribution of the higher order operators are ignorable, i.e. the sum rules are convergent. As a result, the working region for the Borel parameter is found to be $3 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2$. Our numerical results on the mass and decay constant for $D_2^*(2460)$ tensor meson as well as the experimental data on the mass [18] are given in Table 1. The errors quoted in our predictions are due to the variations of both auxiliary parameters and uncertainties in input parameters. From Table 1, we see a good consistency between our prediction and the experimental data on the masses of the $D_2^*(2460)$ tensor meson. Our result on the decay constant can be checked in experiment. Any measurement in this respect and comparison of the result with our predictions can give more information about the nature of the orbitally excited $D_2^*(2460)$ tensor meson.

	Present Work	Experiment [18]
$m_{D_2^*(2460)}$	$(2.50 \pm 0.48) \text{ GeV}$	$(2.4626 \pm 0.0007) \text{ GeV}$
$f_{D_2^*(2460)}$	$(0.0317 \pm 0.0092) \text{ GeV}$	-

Table 1: Values for the mass and decay constant of the $D_2^*(2460)$ tensor meson.

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